INTEGRATION TECHNIQUES

This review of integration techniques is in no way complete. It is vital for your success that you attempt a large number of problems from the text (even more than are assigned). There is no substitute for practice and experience. I hope that this guide helps you organize your studying.

We have 17 integrals we can do in one step (see the next page). If your integral is not on this list, then you need to use our methods. The first thing you should do is look for any possible u-substitutions or simplifications. Then you should try one of our four new methods. These methods are summarized below:

YOUR INTEGRAL

SIMPLICATION OR u-SUBSTITUTION

Products, log's, inverse trig

INTEGRATION BY PARTS

u =   dv =
du =   v =

When choosing u, remember LIPET.

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sin's, cos's, tan's, sec's

TRIG. INTEGRALS

Look at powers

1. Odd cos → u = sin(x)
2. Odd sin → u = cos(x)
3. Even sec → u = tan(x)
4. Odd tan → u = sec(x)
5. Even sin/cos → Half-Angle

For first 4 cases you need:
$\sin^2(x) = 1 - \cos^2(x)$
$\cos^2(x) = 1 - \sin^2(x)$
$\tan^2(x) = \sec^2(x) - 1$
$\sec^2(x) = \tan^2(x) + 1$

The half angle identities:
$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$

If stuck, look at integral table and identities.

$a^2 - x^2$, $x^2 + a^2$, $x^2 - a^2$ and usually a square root

TRIG. SUBSTITUTION

If it has a linear (middle) term, complete square.
(1/2 middle, square, add/ subtract value)

Make substitution:

$a^2 - x^2 \rightarrow x = a\sin(\theta)$
$a^2 + x^2 \rightarrow x = a\tan(\theta)$
$x^2 - a^2 \rightarrow x = a\sec(\theta)$

At end, draw and label the TRIANGLE to get x's back.

You will need all the same trig identities shown in the trig integrals method.

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rational functions where the bottom factors

PARTIAL FRACTIONS

If top power ≥ bot power, then divide.
Then factor bottom, set up partial fractions, solve for A, B, C, ….

Distinct Linear
→ constant for each factor.

Repeated Linear
→ constant for each power of factor from 1 up to the number of times repeated.

Irreducible Quad.
→ Complete the square if needed, then numerator of the factor is Ax+B.

Integration Table

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$	$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$	$\int b^x = \frac{1}{\ln(b)}b^x + C$
$\int \cos(ax)\,dx = \frac{1}{a}\sin(x) + C$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(x) + C$
$\int \sec^2(x)\,dx = \tan(x) + C$	$\int \csc^2(x)\,dx = -\cot(x) + C$
$\int \sec(x)\tan(x)\,dx = \sec(x) + C$	$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$
$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \tan(x)\,dx = \ln\lvert\sec(x)\rvert + C$	$\int \cot(x)\,dx = \ln\lvert\sin(x)\rvert + C$
$\int \sec(x)\,dx = \ln\lvert\sec(x)+\tan(x)\rvert + C$	$\int \csc(x)\,dx = \ln\lvert\csc(x)-\cot(x)\rvert + C$

$$\int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln\lvert\sec(x)+\tan(x)\rvert + C$$

Derivatives Table

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$	
$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(b^x) = b^x\ln(b)$	
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$	$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{x^2+1}$	$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$
$(FS)' = FS' + F'S$	$\left(\frac{N}{D}\right)' = \frac{DN' - ND'}{D^2}$	$[f(g(x))]' = f'(g(x))g'(x)$

Precalculus Facts

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

$\sin^2(x) + \cos^2(x) = 1$	$\tan^2(x) + 1 = \sec^2(x)$	$1 + \cot^2(x) = \csc^2(x)$
$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$	$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$	$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$

$\ln(1) = 0$	$\ln(e) = 1$	$\ln(a^b) = b\ln(a)$	$\ln(ab) = \ln(a) + \ln(b)$
$x^a x^b = x^{a+b}$	$(x^a)^b = x^{ab}$	$\sqrt[n]{x} = x^{1/n}$	$\frac{1}{x^a} = x^{-1}$